VANET communication: a traffic flow approach

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***Abstract*—Macroscopic fluid traffic flow models are used in this paper to predict connectivity behavior within vehicular to vehicular (V2V) networks. The vehicular density determines the radio connectivity in sparse traffic, and may even contribute to radio interference in dense traffic situations. We propose modeling the vehicular density using an approach from physics and engineering known as *traffic flow theory*. It builds models of the interactions between vehicles and road infrastructure similar to fluid or gas flows in pipelines. We have developed analytic solutions for estimating vehicular densities based on a method for solving the traffic flow equations. With very general infrastructure conditions we can find the density evolution at any location and time for any (given) initial conditions. Using such an approach we show how the connectivity, reachability and broadcast capacities of V2V networks can be calculated as applications of this approach. Furthermore we can include the path loss between vehicles and include interference from dense traffic situations in line and non-line of sight situations. We parameterize the method using Bluetooth and IEEE 802.11a radio technologies to show the applicability of such an approach.**

I. Introduction

Traditionally, macroscopic traffic fluid flow[1] models have been used to study the interaction of vehicles on existing road infrastructure in order to optimize traffic flow through cities, towns and highways. In this work we use the same techniques to predict density variations and hence connectivity analysis in Vehicular Ad-hoc Networks (VANET) communications. In general it is difficult to ascertain the connectivity of vehicles in both time and location. The interaction of the vehicles with each other, such as one vehicle changing velocity affecting others, stoppages and highway infra-structure make density prediction, especially ahead in space-time, non-trivial.

We introduce the theory of traffic flow and consider the simplest case, a single lane (scalar) linear traffic flow model. Then we go onto derive the important conservation law. After introducing the vehicle velocity we go on to show how the relationship between vehicle velocity and density gives rise to a non-linear relationship between the density and the traffic flux. We then present two road examples and explain how we solve simple road cases analytically and numerically to highlight the central ideas. We show how traffic flow theory can be used in modeling real road scenarios, how to parametrize the model from real road conditions and importantly how to apply the approach to VANET communications. Finally we give two applications that can be easily constructed from the density estimates we derive from the traffic flow theory.

[1] We denoted flow as a generic traffic term and flux as density $\times$ velocity.

II. Related work

The history of traffic flow lies with fluid dynamicists. Whitam's 1956 publication "Shock waves in the highway" [Whi56] together with the work with Lighthill "On kinematic waves" [LW55] showed that traffic flows can be treated in a similar way as in gas and water flows had been. It was later shown that traffic jams displays sharp discontinuities, whereby a correspondence exists with shock waves seen in nature [Pet72]. A traffic flow approach has been presented in [BNP06], but does not consider a communications setting. In the field of mobile ad-hoc networking Gupta and Kumar present for statically identical randomly located nodes, the throughput obtainable for a source-destination (S-D) pair for a randomly chosen position is $O(\frac{1}{\sqrt{n}})$ [GK00]. Grossglauser and Tse extended this work by including independent node mobility, the average long term throughput per S-D pair can be kept *constant* as the number of nodes per unit area n increases, i.e. $O(n)$ [GT02]. Pishro-Nik et. al provide a general framework to study the fundamental capacity limits of VANETs and show that (indeed) road geometry affects the capacity and connectivity of VANETs and are $\Theta(\frac{1}{n})$ for a single road and $\Theta\left(\frac{1}{\sqrt{n\,ln(n)}}\right)$ for a grid-like road structure for a S-D pair [Hos07]. A probabilistic (percolation) analysis on vehicle clustering and connectivity is presented in [KPD$^+$08].

III. A traffic flow model

A. A simple conservation law

Let ρ be the density of vehicles at a fixed location and time $\rho(x, t)$. The flux of vehicles is denoted q. The number passing a fixed point x at time t is $q(x, t)$. A positive value of $q(x, t)$ indicates the flux is in the direction of increasing x. The flux, density and velocity of the vehicles are related by

$$q(x, t) = \rho(x, t) \cdot u(x, t) \tag{1}$$

It is important to state that the variables above are all functions of *two* variables, both x and t. These and other variables are shown in table I. We now consider the number of vehicles N on a *road segment* $[a, b]$ given by

$$N = \int_a^b \rho(x, t)\, dx \tag{2}$$

which is the integral of the traffic density. The number of vehicles between a and b may change due to the *in* and *out* flows. We allow the number of vehicles crossing the

TABLE I
VARIABLES USED IN THIS WORK

Entity	Notation
Flow	q
Density	ρ
Initial density	ρ_0
Wave speed density	c
Velocity	u
Space	x
Position of vehicle n	x_n
Time	t
Number of vehicle	N
Length of vehicle	L
Road section length	D
Road section	$[a, b]$

boundaries a and b to be variable i.e. $q(a)$ and $q(b)$. The rate of change in the number of vehicles is thus given by

$$\frac{dN}{dt} = q(a,t) - q(b,t). \tag{3}$$

The quantity flowing *into* b and has a negative sign. Although we don't include bidirectional traffic in this paper, by choosing the flux signs for flows in and out of $[a, b]$ we can include flow in either direction. By combining (2) and (3) we obtain

$$\frac{d}{dt}\int_a^b \rho(s,t)ds = q(a,t) - q(b,t). \tag{4}$$

If we consider the conservation law on the interval $[a, x]$ where a is constant and fixed road position and x is an independent variable *anywhere* on the road section, we obtain

$$\int_a^x \frac{\partial \rho}{\partial t}(s,t)ds = q(a,t) - q(x,t). \tag{5}$$

We can replace the b in (4) by x in (5) as the position can be anywhere on the road and differentiating with respect to x we obtain

$$\frac{\partial \rho}{\partial t} = -\frac{\partial q}{\partial x} \tag{6}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0, \quad -\infty < x < \infty, t > 0, \tag{7}$$

with the initial condition

$$\rho(x,0) = \rho_0(x).$$

We must include choose values for the initial condition of a differential equation, as it space-time trajectory will depend on the choice. In numerical methods too and in real road tests, measure/estimate the relevant traffic parameters. Continuing, in simpler notation, (7) can be written

$$\rho_t + q_x = 0. \tag{8}$$

which is the *differential* form of the conservation law. The equation expresses the fact that changes in the number of vehicles are only due to the flow across the boundaries if the right hand side (RHS) is zero. Note that the number of vehicles in the region $[a, b]$ is **not** constant. If that were true, $q(a,t) = q(b,t)$ or $\frac{d}{dt}\int_a^b \rho(x,t)dx$ would be 0 and this is not the case. Thus far we assume that no vehicles enter or leave the road section except at the entrances and/or exits, i.e. there are no side roads. Importantly this *conserves* the number of vehicles in the region and the law derives its name from this principle. Note also there is a velocity associated with the vehicle density as well. That is the density profile can be observed to move when for example a lead vehicle in a formation brakes, and a change in the vehicle separation appears to move backwards.

B. From conservation to flow

Now we need to be able to solve the conservation law (8) for different road conditions. We start with the simplest linear case. It describes an initial wave profile traveling with a constant velocity c along a road. A profile in this context means an initial measurement or model of the traffic at the entrance of a road section.

$$\rho_t + cq_x = 0, \quad x \in \mathbb{R}, \quad t > 0 \tag{9}$$

$$\rho(x,0) = \rho_0(x) \tag{10}$$

Going back to equation (1) we know that the flux is related to the density $q(x,t) = c\rho(x,t)$ where the velocity of the vehicles in this case is considered as a constant (c). The solution to this particular initial value problem is straightforward, the wave profile is the original one shifted forward in space and time for a positive c, i.e. $\rho(x,t) = \rho(x - ct)$. That is for any initial profile after some time, the initial profile would be the same across the whole road section.

C. Density dependency

It is reasonable to expect the velocity of a vehicle to be a function of the local density i.e. $u = u(\rho)$, in fact it decreases monotonically with increasing density, the more vehicles in a region, the lower the average velocity. This is intuitive as vehicles must slow down in higher density traffic and is shown in the left plot of figure 1. Note also the flux q of a traffic flow will be a function of the density as well since $q = u \cdot \rho$

$$q = q(\rho), \quad q(\rho) = \rho(1-\rho). \tag{11}$$

Since we defined the flux as the product of the velocity and the density. When the density is at a minimum ($\rho = \rho_{min} = 0$) the flux is zero, since there are no vehicles to constitute a flow. Conversely, when the density is maximum ($\rho = \rho_{max}$) vehicles are queued up back to back i.e. no movement, and again there is no flux. The two cases are shown in the center plot of figure 1 with a maximum where there is some density and some velocity. Continuing on, we have

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(q(\rho)) = 0 \tag{12}$$

and by the chain rule

$$\frac{\partial q}{\partial x} = \frac{dq}{d\rho}\frac{\partial \rho}{\partial x} \tag{13}$$

The conservation law now becomes

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0 \tag{14}$$

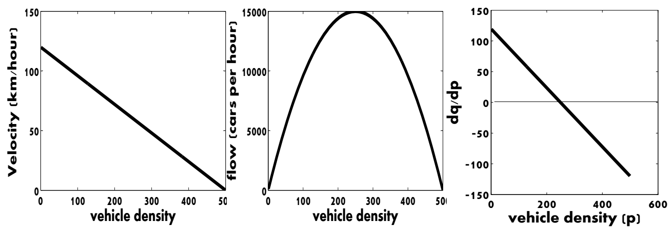

Fig. 1. The fundamental diagram of road traffic. **Left** the velocity as a function of density. **Center** the flux (vehicles/hr) as a function of density (vehicles/km). **Right** the derivative $\frac{dq}{d\rho}$ wrt density.

$$\frac{\partial \rho}{\partial t} + c(\rho)\frac{\partial \rho}{\partial x} = 0 \tag{15}$$

The conservation law is now non-linear due to the wave velocity's dependence on the density ρ. We will now see how to solve the linear and non-linear cases with a mechanism which converts the partial differential equation into an ordinary differential equation, called the method of characteristics.

D. The method of characteristics

In the linear case we had

$$\frac{\partial \rho}{\partial t} + c\frac{\partial \rho}{\partial x} = 0. \tag{16}$$

Looking at the solutions of equation (16) along a specially chosen line $x = x(t)$ gives solutions $\rho(x(t), t)$. Differentiating this with respect to t we obtain

$$\frac{d\rho}{dt} = \rho_x \frac{dx}{dt} + \rho_t \frac{dt}{dt} \tag{17}$$

which is known as the total differential. Should $\frac{dx}{dt}$ in (17) equal c in (16) then

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial x} \cdot c + \frac{\partial \rho}{\partial t} = 0 \tag{18}$$

along the line $x(t)$. This means that ρ is constant along any such line for an observer moving with speed c. By integrating $\frac{dx}{dt} = c$ with respect to t we obtain $x = x_0 + ct$. The lines of constant density at different constants x_0's are called characteristics and are shown in the right hand plot of figure 2. The characteristic gradient is $+\frac{1}{c}$ for positive gradients. Note that the gradient is the reciprocal of c as we using t versus x.

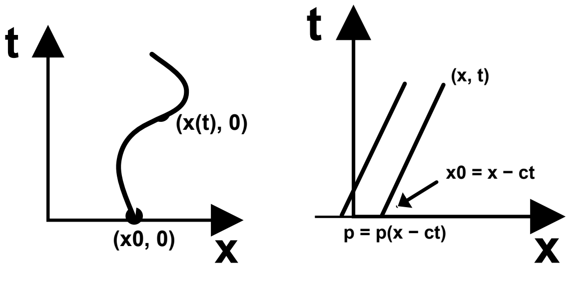

Fig. 2. **Left:** Specially chosen curve $(x(t), t)$ in the xt plane where the density is constant. **Right:** Characteristics $x = x_0 + ct$ of the equation $\rho_t + c\rho_x = 0$ equation (16) .

E. The linear case

As an example consider the following conservation law:

$$\rho_t + 4\rho_x = 0, \quad -\infty < x < \infty, t > 0$$

With wave profile, i.e. the initial conditions

$$\rho(x, 0) = \text{arctan(x)}$$

Along a curve $(x(t), t)$, the derivative of $\rho(x(t), t)$ is

$$\frac{d}{dt}\rho(x(t), t) = \rho_x(x(t), t)\frac{dx}{dt} + \rho_t(x(t), t)$$

Picking $x(t)$ to satisfy:

$$c = \frac{dx}{dt} = 4, \quad x(0) = x_0$$

so $\rho(x(t), t)$ has a constant value along $x = 4t + x_0$. At any point (x, t) the characteristic line through this point extends back to $(x_0, 0)$ on the x-axis where $x_0 = x - 4t$. Since ρ is constant *along this characteristic*, the value of ρ at (x, t) is:

$$\rho(x, t) = \rho(x_0, 0) = \text{arctan}(x_0) = \text{arctan}(x - 4t)$$

The solution of the initial value problem is a traveling wave with profile arctan(x) moving with velocity 4.

F. The non-linear case

We now turn to the non-linear case, which has a dependence on ρ. *But* whatever the characteristic ends up being, its value will still be constant along the curve $(x(t), t)$

$$\rho_t + c(\rho)\rho_x = 0, \quad \rho(x, 0) = u_0(x), \quad -\infty < x < \infty, t > 0$$

The characteristic starting at $(x_0, 0)$ is found by solving

$$\frac{dx}{dt} = c(\rho(x, t)), \quad x(0) = x_0.$$

$$\begin{aligned}\frac{d}{dt}\rho(x(t), t) &= \rho_t(x(t), t) + \rho_x(x(t), t)\frac{dx}{dt} \\ &= \rho_t(x(t), t) + c(\rho(x(t), t))\rho_x(x(t), t) = 0\end{aligned}$$

The value of ρ along a characteristic is **still** constant. The usefulness of characteristic depends on this property in converting PDEs to ODEs.

G. A traffic light example

Traffic is lined up behind a red traffic light, positioned $x = 0$, bumper to bumper implies $\rho = \rho_{max}$ for $x < 0$, and there is no traffic ahead of the light $\rho = 0$ for $x > 0$. We need to solve the following:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial \rho}\frac{\partial \rho}{\partial x} = 0$$

with the initial conditions:

$$\rho(x, 0) = \begin{cases} \rho_{max} & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases}$$

Using characteristics, the density propagates at a velocity $dq/d\rho$. If the density ρ remains constant, then the density moves with constant velocity. The characteristics are straight lines in the xt plane:

$$x = \frac{dq}{d\rho}(\rho)t + k$$

Each characteristic may have a different integration constant k. Looking at all the characteristics that intersect the initial data at $x > 0$, there $\rho(x, 0) = 0$. Thus $\rho = 0$

$$\frac{dx}{dt} = \frac{dq}{d\rho}(\text{at } \rho = 0) = u(0) = u_{max}$$

which means:

$$x = u_{max} + x_0(x_0 > 0)$$

And it is possible to calculate the waiting time for the n^{th} vehicle as

$$t = \frac{(n-1)L}{-\rho_{max}u'(\rho_{max})}$$

$$u'(\rho_{max}) \approx \frac{\delta u}{\delta \rho} = \frac{-6 \text{ km/hr}}{60 \text{ vehicles/km}} = -0.1\frac{km^2}{\text{car} \cdot \text{hour}}$$

For each vehicles behind the light, the waiting time is

$$t = \frac{L}{-\rho_{max}u'(\rho_{max})} = \frac{1}{-\rho_{max}^2 u'(\rho_{max})} = \frac{1}{0.1 \cdot (225)^2}$$

which is $\approx$ 0.7 secs. This shows us how we can calculate traffic densities in space-time.

H. Shockwaves

An important concept in all fluid flow work is that of shockwaves. The occur in road situations where differing densities exist along a road section and the characteristics may become multivalued. This occurs when faster lighter moving traffic "catches up" with slower moving higher density traffic ahead of it. An example of the how the velocity of vehicles is related to the density waves can be seen in the New Scientist publication video `http://www.youtube.com/watch?v=Suugn-p5C1M`. The speed of the resulting density waves is always less than the velocity of the vehicles, as seen by:

$$c = \frac{dq}{d\rho} = \frac{d}{d\rho}(\rho u) = u + \rho\frac{du}{d\rho} \tag{19}$$

$$c < u \quad \text{only if} \quad \frac{du}{d\rho} \leq 0 \tag{20}$$

This implies that drivers always react to changes in density ahead of their current position and not behind. Vehicles in a congested region have large densities in their vicinity and consequently must move slowly. This is akin to sudden large densities occurring in "flash" traffic situations.

IV. A General Numerical Technique

To solve non-linear PDEs (with shockwaves) we need to consider the integral conservation law rather than the differential law. The reason being that numerical solutions may not be found at all points along the road. Note it is generally very difficult to predict shockwaves as they depend on the initial density, road parameters and where the changes in density profiles occur in space-time. One form of the integral conservation is given by (21).

$$\int_a^b \rho(x,t_2)dx = \int_a^b \rho(x,t_1)dx \; - \int_{t_1}^{t_2} f(\rho(b,t))dt \; + \int_{t_1}^{t_2} f(\rho(a,t))dt \tag{21}$$

A. A simple numerical algorithm

We need to be able to calculate the density over the road section. Since partial derivatives are limits of difference quotients, forward and backward approximations can be calculated numerically, as shown in equations (22) and (23) respectively. They are derived directly from the integral form of (21). Note, we may need the forward and backward forms for numerical stability.

$$\frac{\partial\rho}{\partial t_n} \approx \frac{\rho(x_i, t_n + k) - \rho(x_i, t_n)}{k}, \frac{\partial\rho}{\partial x_i} \approx \frac{\rho(x_i + h, t_n) - \rho(x_i, t_n)}{h} \tag{22}$$

$$\frac{\partial\rho}{\partial t_n} \approx \frac{\rho(x_n, t_i) - \rho(x_n, t_i - k)}{k}, \frac{\partial\rho}{\partial x_i} \approx \frac{\rho(x_i, t_n) - \rho(x_i, t_n - h)}{h} \tag{23}$$

The above form can be manipulated algebraically to give

$$\rho(x_n, t_i + k) = (1 - \frac{ck}{h})\rho(x_n, t_i) + \frac{ck}{h}\rho(x_n - h, t_n). \tag{24}$$

We use the x_n and t_i values separated by h and k in the numerical aspects (next section) of this work, by employing Godunov's algorithm, which we do not explain in more detail due to space constraints.

B. Traffic flow parameters

We will consider the traffic at the start of a 2km road. We specify the maximum density ρ_{max} of 500 (2000 / 4) and the maximum velocity of 120 km/hr. For the cell size we selected 20m ($\triangle x$) to match Bluetooth's coverage which is also five times the length of an average family saloon. These values simply reflect the road distances and physical connectivity relationships with respect to wireless communication and can of course be selected as needed. The inflow to the road section will be of two types from this point in the paper **i)** vehicles behind a stoppage. A condition such as a traffic light at the beginning or end of a road stretch is known as a boundary condition. We consider waiting times of 30, 60 and 120 seconds. At an arrival rate of 1 vehicle per km there would be 30, 60 and 120 vehicles respectively behind the lights. **ii)** a more constant stream of heavy ($0.8{\cdot}\rho_{max}$), medium ($0.5{\cdot}\rho_{max}$) and light ($0.2 \cdot \rho_{max}$) stream entering the road section at a. We have introduced a small amount of randomness into this stream to reflect that the vehicles are not exactly equally spaced.

C. Examples

Examples of the density variations (over distance) are shown in figures 3 and 4. Figure 3 illustrates a situation where vehicles are backed up behind a traffic light and five seconds later, the light turns green. The initial density profile changes from the initial step form on the left to the one shown on the right, as vehicles in front can accelerate away whilst those behind must start slower. In figure 4, we have the opposite case, the density changes from low to high as modeled by the function $arctan(\cdot)$, in this case the faster moving waves catch up to the slower moving ones in front and a step density profile develops. $arctan(\cdot)$ is such a function that represents such a smooth transition from low to high densities as shown to illustrate the earlier example. It is important to point out any function (analytic or empirical) may be used.

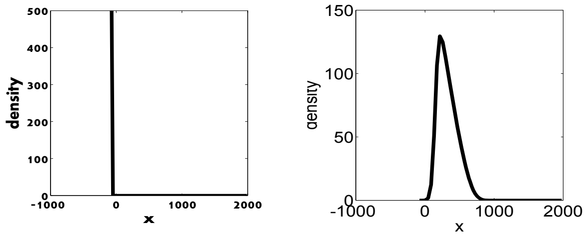

Fig. 3. **Left:** Initial step density **Right:** 5 seconds later.

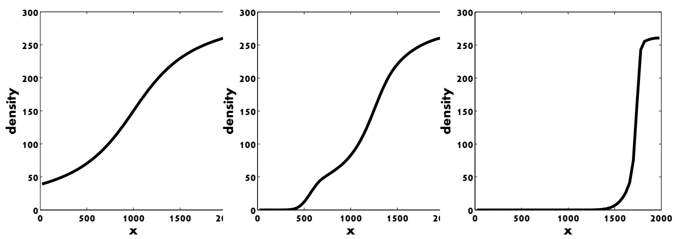

Fig. 4. **Left:** Initial low-high density **Middle/Right:** 5 and 15 seconds later

V. V2V COMMUNICATIONS

A. Information propagation

We now show *how* to use traffic flow density calculations in VANET communications. Three applications have been chosen for this first part, based on [Du08].

1) The level of **connectivity** is defined as the relative time during which adjacent cells are connected.
2) **Reachability** defined as the probability that every two vehicles in the network are connected (clearly coverage dependent)
3) **Broadcast capacity**, defined as the maximum number of successful concurrent transmissions over the road.

B. Generic algorithm

$\bar{P} \leftarrow \frac{P}{\rho_{max}}$ $\triangleright$ Normalize P wrt the maximum density
$N_i \leftarrow \bar{P} \cdot \frac{d_{cov}()}{L}$
$\tilde{P} \leftarrow round(N_i)$ $\triangleright$ Obtain the no. of vehicles per cell
for all $\triangle t$ **do**
  if ($\exists$ active cells) **then**
    Partially connected
  end if
  if $(\rho \geq 1)$ **then**
    reachability $\leftarrow$ count adjacent cells
  end if
  broadcast capacity $\leftarrow$ max. of 2 adjacent cells
end for

C. Connectivity

Using the algorithm to process the density matrix we can obtain connectivity results in table II. Unsurprisingly the coverage is higher for 802.11a, however the actual percentage coverage have been calculated using the method described. We emphasize the application of the theory is simple, once the density matrix is obtained. We will use the connectivity later on too. A connectivity analysis is important in most mobile ad-hoc networks.

TABLE II
CONNECTIVITY FOR STEP & CONSTANT INITIAL CONDITIONS

Stop time	Duration (seconds)	Bluetooth coverage	802.11a coverage	Traffic flow	Proportion to to ρ_{max}	Bluetooth coverage	802.11a coverage
Short	30	3%	24%	Light	0.2	4%	26%
Med.	60	6%	32%	Medium	0.5	17%	39%
Long	120	12%	45%	Heavy	0.8	56%	74%

D. Reachability

In terms of reachability a critical amount of traffic is needed for the clusters to form for 802.11a and Bluetooth as shown in table III. Naturally, the longer the traffic is queued, clusters form more easily as is also the case for denser traffic entering the road section under consideration. Reachability, particularly the maximum and minimum values are important in Delay Tolerant Network (DTN) applications, even in V2V situations.

TABLE III
REACHABILITY FOR STEP & CONSTANT INITIAL CONDITIONS

Stop time	Duration (seconds)	Bluetooth reach.	802.11a reach.	Traffic flow	Proportion to to ρ_{max}	Bluetooth reach.	802.11a reach.
Short	30	1%	24%	Light	0.2	0%	25%
Med.	60	2%	32%	Medium	0.5	33%	37%
Long	120	10%	45%	Heavy	0.8	70%	73%

E. Broadcast capacity

The value for broadcast capacity are shown in table IV. The broadcast capacity of the network is important when trying to reach a set of nodes simultaneously. In the case where only one node has particular information to disseminate, the broadcast capacity will be utter importance. Note, the information may be data or control information, for example to synchronize an action across a set of nodes.

VI. APPLICATION: RADIO TECHNOLOGIES

To show one application of this work in a radio context, we use data as given in in table V.

TABLE IV
BROADCAST CAPACITY FOR STEP & CONSTANT INITIAL CONDITIONS

Stop time	Duration (seconds)	Bluetooth broadcast	802.11a broadcast	Traffic flow	Proportion to to ρ_{max}	Bluetooth broadcast	802.11a broadcast
Short	30	0.8%	4%	Light	0.2	1%	15%
Med.	60	1.2%	12%	Medium	0.5	2.5%	27%
Long	120	2%	15%	Heavy	0.8	4%	33%

TABLE V
TRANSMISSION PROPERTIES FOR TWO POPULAR TECHNOLOGIES

Technology	Subclass	Typical Range (meters)	Coverage (meters)	Transmit power (dBm)	Transmit power (mW)	Antenna gains G_t and G_r (dBi)
Bluetooth	Class 1	10	20	4	2.5	3
802.11a	Outdoor	120	240	27	500	5

a) Path loss: The path loss, or attenuation, is an important factor in all wireless communication (25),

$$P_r(dB) = -10\log_{10}\left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^2(d+L)^\gamma}\right]. \qquad (25)$$

P_t is the transmission power, G_t is the gain of the transmitter, G_r the gain of the receiver, λ the wavelength of the carrier, d is the distance between transmitter and receiver, γ is known as the path loss exponent which is related to the vehicle density. Where there are two vehicles and no more (i.e. LOS) within a coverage distance means we can calculate the received signal from equation (25) directly. Where however, there are intervening vehicles we must use a higher path loss exponent (γ). In received signal estimation a value of 2 is often chosen for the LOS case. In the case of none-LOS where vehicles attenuate signals, the literature [OBB09] suggests using values of in the range $[2.5, 4.8]$. Figure 5 shows the path losses for a LOS and NLOS case derived from light and heavy initial densities over a 150m road section.

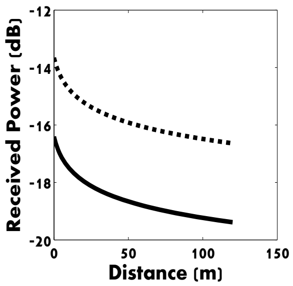

Fig. 5. Path loss for LOS (upper) and NLOS (lower) V2V communication

b) Interference: A standard interference model for wireless communication is is known as the Signal to Interference plus Noise Ratio (SINR) model [GK00]. The model depends on the strength of the received signal. The other vehicles contribute to the interference as well as the ambient noise level (N). For a sender-receiver pair $k_i = (s_i, r_i)$, they use $I_{r_i}(s_j) = P_{r_i}(s_j)$ for other senders s_j sending at the same time as s_i to model the signal power transmitted by s_j is seen as r_i as interference. The total interference $I_{r_i}(K)$ experienced by a receiver r_i is the sum of the interference power by the set K of nodes transmitting at the same time, minus the sender of course. N is the ambient noise power level. Then, r_i receives s_i transmission iff:

$$SINR(k_i) = \frac{P_t r_i(s_i)}{N + I_{r_i}(L)} = \frac{\frac{P_t(s_i)}{d(s_i,r_i)^\gamma}}{N + \sum_{j\neq i}\frac{P_t(s_j)}{d(s_j,r_i)^\gamma}} \geq \alpha \quad (26)$$

Where $\alpha \geq 1$ is needed for successful reception. All we need to know is the k_i pairs within coverage distance. This we have from the connectivity matrix P. Therefore it is straightforward to extract the pairs than can reach other. γ will depend on the number of intervening obstacles, but that is simply $\tilde{P}$ in the method we have presented in section V-B of this paper.

VII. CONCLUSIONS

The focus of this paper has been very much on the application of *traffic flow* theory to VANET connectivity analysis. Its greatest strength is the ability to prediction of density changes in space and time. Importantly this work places little restrictions on the distribution of vehicles. We have shown, that once the densities have been found it is relatively straightforward to calculate other features of VANETs. We have chosen to show the connectivity, reachability and broadcast capacity but also line-of-sight and non line of sight cases and path losses including interference from dense traffic. This is not easy to include with a microscopic modeling approach. This work can also be extended to include roundabouts, intersections and even the case where traffic leaves and enters a road section by considering the right hand side of (8) as non-zero. We acknowledge the Portuguese Foundation for Science and Technology under scholarship number SFRH/BPD/65961/2009.

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