A traffic flow analysis of VANET communication

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Better said as "SICS"

Disclaimer!

The big picture

Motivation for this work

- Internet is here, so are vehicles!
- Vehicular to vehicular (V2V) and vehicular to infrastructure (V2R) touted as the next large major deployment
- Communications protocols have been deployed (via verde) and are actively being developed
- The deployed driven by real needs
  - Safety (inform of crash ahead)
  - Management (heavy loads ahead)
  - GPS augmentation (position known, but density isn't)
- Cellular access (3G) is not always sufficient:
  - Path of communications can be too slow for some applications
  - Vehicle $\rightarrow$ infrastructure to computing center $\rightarrow$ redistribution
  - Spectrum $\rightarrow$ message redistribution costly (Hz, Euro)
- Problem combines application and theory (cross disciplinary)
- Relatively novel
- Pretty complete from problem formulation to implementation

Outline

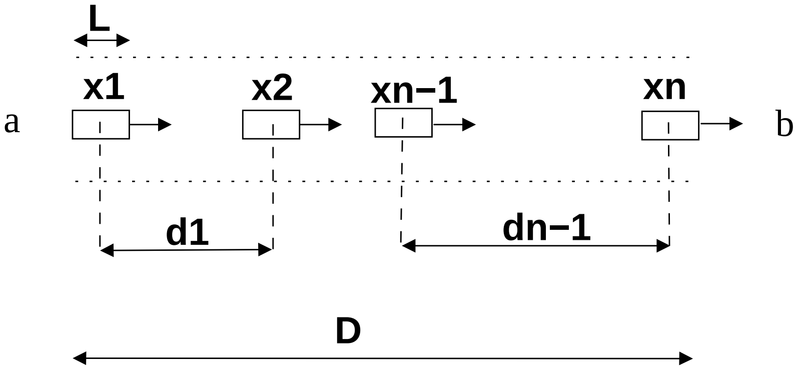
- Part I:
  - Main variables
  - Differential conservation law of PDEs
  - The method of characteristics I
  - Nonlinear first order differential equations
  - The method of characteristics II
  - Shockwaves
  - Integral conservation law
  - Numerical solutions of non-linear PDEs
  - Godunov's algorithm
  - Parametrization of model
- Part II:
  - Radio environment
  - Object shadowing
  - Example of connectivity results
  - Cluster analysis
- Part III:
  - Further work (incl. sidestepped mathematics)
  - Conclusions
  - Small demo

Part I: Traffic flow

Assumptions

- Vehicles are the same $\rightarrow$ cars of 4m in length
- The vehicle position is its center (antenna is there anyway)
- Vehicles cannot overtake

Traffic flow variables



The static density is

$$\rho = \frac{\sum_{n=1}^{N} n}{D},$$

i.e. cars per km.

Traffic flow entities

- ρ is the density of vehicles per unit length
- q is the flow (or flux) of vehicles per unit time
- u is the velocity of vehicles
- ρ, q and u are functions of 2 variables location and time (x, t)
- At time t the net rate flowing into D is rate flowing into $x = a$ minus the rate that is flowing out at $x = b$
- $Aq(a, t)$ if we consider a surface A, we use a 1 dimensional model ($A = 1$)
- The flow, density and velocity of the vehicles are related:
- $q(x, t) = \rho(x, t) \cdot u(x, t)$ or $u(x, t) = q(x, t)/\rho(x, t)$
- N is the number of cars in the road interval
- Conservation law: Through the entrance or exit *only* may cars enter or leave

A linear conservation law I

The number of vehicles (N) on a *road segment* $[a, b]$ given by

$$N = \int_a^b \rho(x, t)\, dx$$

The rate of change in the number of vehicles on the stretch is given by

$$\frac{dN}{dt} = q(a, t) - q(b, t).$$

By the conservation law these should be equal

$$\frac{d}{dt} \int_a^b \rho(s, t) ds = q(a, t) - q(b, t).$$

The conservation law on [a, x] where a is constant and x is an independent variable

$$\int_a^x \frac{\partial \rho}{\partial t}(s, t) ds = q(a, t) - q(b, t).$$

Replace b by x as the position can be anywhere on the road and differentiating wrt x

$$\frac{\partial \rho}{\partial t} = -\frac{\partial q}{\partial x} = 0, \quad -\infty < x < \infty, t > 0.$$

(The integrand is a continuous function of x, and since the above holds for all intervals in D then the integrand must vanish identically, the argument based on the the large box method)

$$\rho_t + q_x = 0.$$

A linear conservation law II

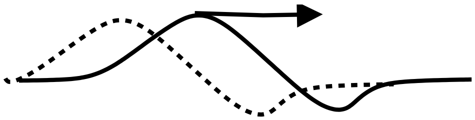
- This is the *differential* form of the conservation law
- Physically, the density changes over time at a fixed point, whilst the flow at a fixed time depends on the location
- The number of vehicles in the region $[a, b]$ is **not** constant. If that were true, $q(a, t) = q(b, t)$ or $\frac{d}{dt} \int_a^b \rho(x, t)dx$ would be 0 and this is not the case.
- Until now, the equation is linear and depends on only two variables, the position x and the time t.
- The initial condition defines a function of the traffic density at time 0 with a function $\rho_0(x)$, as we will see later.
- The conservation equation assumes that ρ and q are continuous, however when they are not, another class of solutions may be found known as *weak* solutions.

Traveling waves

Fundamental mathematical representation of a wave is

$$\rho(x,t) = f(x - ct)$$

f is the function of a single variable and c is nonzero constant. If c is positive then the profile of the $\rho(x,t)$ moves in the positive x direction at speed c.



The profile does not change and is known as a traveling wave.

The method of characteristics

A characteristic is a curve in spacetime along which density propagates.

$$\rho_t + cq_x = 0, \quad \rho(x,0) = u_0(x), \quad -\infty < x < \infty, t > 0 \tag{1}$$

Using the constitutive relation $q_x = c\rho_x(x,t)$ along a chosen line $x = x(t)$ in the xt plane starting from $(x_0, 0)$ the rate of change of $\rho(x(t), t)$ wrt t

$$\frac{d}{dt}\rho(x(t),t) = \rho_t(x(t),t)\frac{dt}{dt} + \rho_x(x(t),t)\frac{dx}{dt}$$

$$\frac{d\rho}{dt} = \rho_t + \frac{dx}{dt}\rho_x \tag{2}$$

Comparing equations 1 and 2 then $\frac{dx}{dt}$ equals the constant c and importantly $\frac{d\rho}{dt} = 0$ along the line $x(t)$

$$\frac{d\rho}{dt} = \rho_t + c\rho_x = 0 \tag{3}$$

The change in density is 0 along $x(t)$ and by integrating with respect to t, ρ will be a constant. The lines of constant density are called *characteristics*.

$$\frac{dx}{dt} = c$$

then by integrating again we obtain

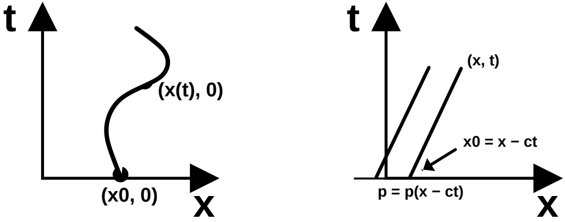
$$x = x_0 + ct \quad \text{or} \quad x_0 = x - ct$$

which is the solution to the traveling wave form from the previous slide

$\rho(x,0) = \rho_0(x), \rho(x,t) = f(x - ct)$.

Characteristics illustration

- The right hand plot illustrates a "specially" selected curve in the xt plane in which the density remains constant.
- implies the density along this curve is the same and can be found from the initial condition $\rho_0(x_0)$.
- In the traffic flow literature the gradient of the characteristics is referred to as their "speed".
- Their gradient is $\frac{1}{c}$ for positive gradients and $-\frac{1}{c}$ for negative cases.
- The gradient is the reciprocal as we using t versus x rather than x versus t.



- Given the fact that ρ is constant along the lines $x = ct + x_o$ we can construct the density at any time
- Alternative thinking: The density is constant, seen by an observer moving in a particular path and speed
- An example is next

Example

As an example consider the following conservation (advection) law:

$$\rho_t + 4\rho_x = 0, \quad -\infty < x < \infty, t > 0$$

With wave profile, i.e. the initial conditions

$$\rho(x, 0) = \text{arctan(x)}$$

Picking $x(t)$ to satisfy:

$$c = \frac{dx}{dt} = 4, \quad x(0) = x_0$$

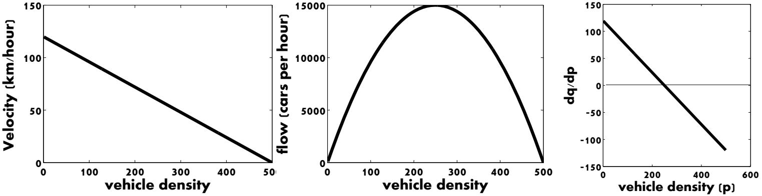
so $\rho(x(t), t)$ has a constant value along $x = 4t + x_0$. At any point (x, t) the characteristic line through this point extends back to $(x_0, 0)$ on the x-axis where $x_0 = x - 4t$. Since ρ is constant *along this characteristic*, the value of ρ at (x, t) is:

$$\rho(x, t) = \rho(x_0, 0) = \text{arctan}(x_0) = \text{arctan}(x - 4t)$$

The solution of the initial value problem is a traveling wave with profile arctan(x) moving with velocity 4.

Traffic flow realities

In practice the velocity of a vehicle is a function of the number of cars around it i.e. $u = u(\rho)$. It has been shown empirically that the velocity decreases monotonically with increasing density (left)



The flow is will also become a function of the density, a form that is typically used is

$$q(\rho) = \rho(1 - \rho).$$

When the density is at a minimum ($\rho = \rho_{min} = 0$) the flow is zero $q(\rho) = 0$, no vehicles for a flow, density a maximum ($\rho = \rho_{max}$) again no flow $q(\rho) = 0$, because cars are stacked bumper to bumper (middle).

Non-linear characteristics

We can now introduce the fact the flow (and velocity) depend on the density.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(q(\rho)) = 0 \tag{4}$$

then by the chain rule:

$$\frac{\partial q}{\partial x} = \frac{dq}{d\rho}\frac{\partial \rho}{\partial x} \tag{5}$$

The conservation law now becomes

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0 \tag{6}$$

$$\frac{\partial \rho}{\partial t} + c(\rho)\frac{\partial \rho}{\partial x} = 0 \tag{7}$$

We have a form as in the non-linear case, but with dependence on ρ *But* whatever the characteristic ends up being, its value will still be constant along the curve $(x(t), t)$

$$\rho_t + c(\rho)\rho_x = 0, \quad \rho(x,0) = u_0(x), \quad -\infty < x < \infty, t > 0$$

The characteristic starting at $(x_0, 0)$ is found by solving

$$\frac{dx}{dt} = c(\rho(x,t)), \quad x(0) = x_0.$$

$$\frac{d}{dt}\rho(x(t),t) = \rho_t(x(t),t) + \rho_x(x(t),t)\frac{dx}{dt}$$
$$= \rho_t(x(t),t) + c(\rho(x(t),t))\rho_x(x(t),t) = 0$$

The value of ρ along a characteristic is **still** constant.

Non-linear characteristics

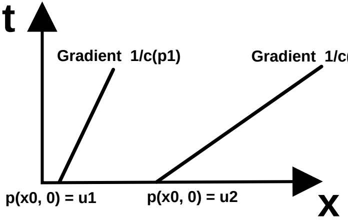
Since the $\rho(x,t)$ has the constant value $\rho_0(x_0)$ along the characteristic starting at $(x_0, 0)$ the initial value problem

$$\frac{dx}{dt} = c(\rho_0(x_0)), \quad x(0) = x_0$$

can be solved

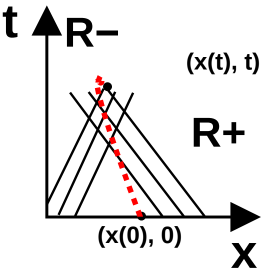
$$x = c(\rho_0(x_0))t + x_0$$

the characteristics are are lines, but they are not parallel since the gradient depends on the value of ρ at the initial point.



Shockwaves

- Shock waves can be seen as abrupt density changes
- Effects are dramatic e.g. explosions but we still need to be able to handle them in this work
- Where differing densities exist at places along a road the characteristics may become multivalued
- The relative differences in the density determine how fast shock waves form
- The method of characteristics can construct a solution but only up to the point where the function becomes multi-valued



Density ahead and behind

- It can be shown that the velocity of the shockwaves is always less than the mean speed of the vehicles themselves.

$$c = \frac{dq}{d\rho} = \frac{d}{d\rho}(\rho u) = u + \rho \frac{du}{d\rho}$$

$$c < u \quad \text{only if} \quad \frac{du}{d\rho} \leq 0 \tag{8}$$

- Means that drivers always react to changes in density ahead of their current position and not behind.
- Vehicles in a congested region have large densities in their vicinity and consequently must move slowly.
- Vehicles behind travel faster until they move into a higher density region and must slow down once reaching it.
- Hence shocks (multi valued functions) develop
- An example of velocity of vehicles and density waves next
- `http://www.youtube.com/watch?v=Suugn-p5C1M`.
- (Note the cars move at 30km and the shockwave at 20km)

Integral form of conservation law

Has the form

$$\frac{d}{dt}\int_a^b \rho(s,t)ds = q(a,t) - q(b,t).$$

if ρ and q are assumed to have continuous first derivatives then the form above can be written as

$$\int_a^b \rho_t(x,t)dt = -\int_a^b q_x(x,t)dt$$

$$\int_a^b (\rho_t(x,t)dt + q_x(x,t))dx = 0$$

This form is more applicable when the density and flux are not smooth functions (continuously differentiable)

A general numerical technique

- We need a general technique to solve the conservation hyperbolic equations presented thus far.
- We need to consider the integral conservation law when constructing numerical solutions
- The reason being that solutions to the differential form may not be found at all points (partial derivatives don't exist)
- This happens physically when the traffic flow develops shocks
- The integral conservation law can be written as

$$\int_a^b \rho(x,t_2)dx = \int_a^b \rho(x,t_1)dx \quad - \int_{t_1}^{t_2} f(\rho(b,t))dt \quad + \int_{t_1}^{t_2} f(\rho(a,t))dt$$

A simple numerical algorithm

Partial derivatives are limits of difference quotients, so forward (9) and backward (10) approximations can be calculated

$$\frac{\partial \rho}{\partial t} \approx \frac{\rho(x, t+k) - \rho(x, t)}{k}, \frac{\partial \rho}{\partial x} \approx \frac{\rho(x+h, t) - \rho(x, t)}{h} \tag{9}$$

$$\frac{\partial \rho}{\partial t} \approx \frac{\rho(x, t) - \rho(x, t-k)}{k}, \frac{\partial \rho}{\partial x} \approx \frac{\rho(x, t) - \rho(x, t-h)}{h} \tag{10}$$

resulting in

$$\rho(x, t+k) = (1 - \frac{ck}{h})\rho(x, t) + \frac{ck}{h}\rho(x-h, t).$$

- $\frac{ck}{h}$ is called the Courant-Friendrichs-Levy (CFL) condition and is important for stability and convergence in many types of numerical hyperbolic PDEs algorithms
- Since density propagates along characteristics, one important aspect is numerical stability and convergence of the numerical solution.

Godunov's algorithm

- Godunov's method is a numerical algorithm for solving PDEs.
- x and t values separated by h and k use a grid
- Well suited to boundary conditions, both in the mathematical and traffic senses!
- The integral form is used rather than the differential one for numerical stability (partial derivates may not exist).
- Define a spatial grid in with a mesh size of $\triangle x$. This has relevance for us, as we need a fixed distance to calculate the density changes.
- A function v defined on the grid

  $$v_m^j = v(t_j, x_m) \quad j \in \mathbb{Z} \quad \text{and} \quad m \in \mathbb{N}$$

  and at every time step $(\triangle t,\ v_j)$ is calculated

  $$v_m^{j+1} = v_m^j - \frac{\triangle t}{\triangle x}(Q_{j-1/2} - Q_{j+1/2}),$$

- $Q_{j+1/2}$ is the approximation to the flow $Q(x + \triangle x/2)$ averaged over the time interval $(t, t + \triangle t)$.
- Interacting waves from adjacent cells use a limited time interval $\triangle t$.

Traffic flow parameters

- Length of an average family saloon 4m
- We will consider the traffic at the start of a 2km road.
- We specify the maximum density p_{max} of 500 (2000 / 4)
- Maximum velocity 120 km/hr
- The time dimension is usually determined via the fundamental traffic law, by obtaining a maximum velocity (hence minimum time)
- The size of the cells in terms of distance was selected at 20m
- Matches Bluetooth's coverage and is x5 average car length
- Tests conducted
  - Scenario 1: Backlogged cars behind a red traffic light
  - Scenario 2: More or less constant traffic along a highway
  - Scenario 3: Low to high density moving traffic
  - Scenario 4: Measured traffic (underway)
  - Use low, medium and high densities relative to ρ_{max}

Modeling the initial (random) conditions

- Intercar spacing needs to be captured: uniform, exponential...
- Use uniform distribution to generate other distributions
- To generate other distributions the CDF should be invertible
- For the exponential:

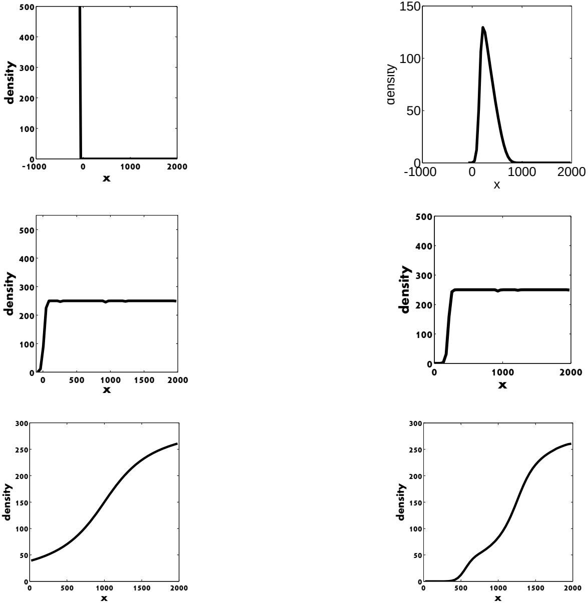
$$x_i = \frac{1}{p} log(u_i) \quad \text{AND} \quad \tilde{x}_i = \frac{1}{p} log(\tilde{u}_i)$$

  Where p is the mean of the distribution.

Antithetic technique

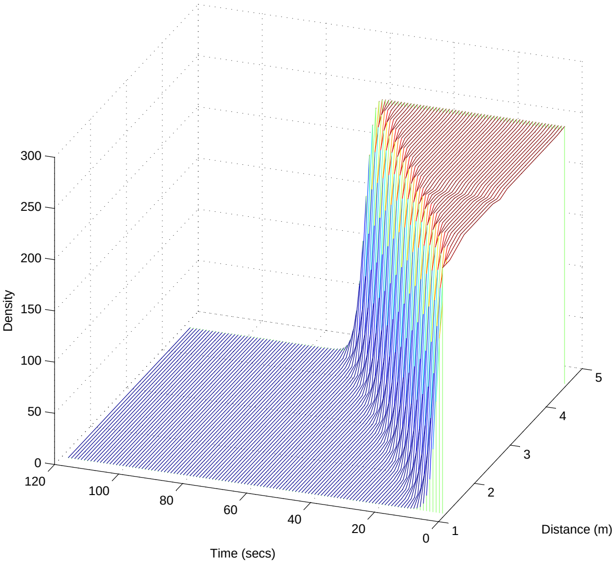
- The problem is variations in RVs are too large
- One method is to use *antithetic* variates
- Generate a *pair* of random variables with negative covariance
  - Difference between covariance and correlation?
- The key idea is to exploit $Cov[u, \tilde{u}] < 0$ in $Var(X + Y)$
- By using U and $1 - U$, the samples are not IID any longer
- Variance reduction depends on i, however $1/50$ is achievable

Density evolution examples



- Initial densities at $t = 0$ and their evolution at $t = 5$
- Note small variance in uniform case

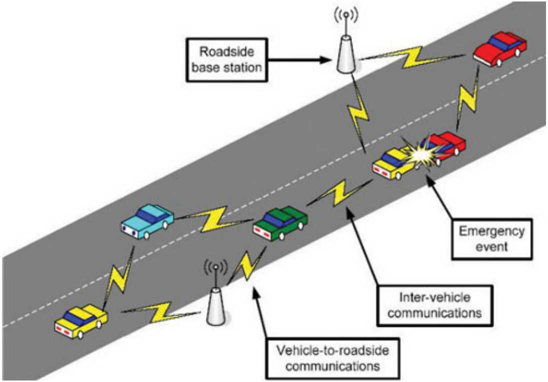
Sexier plot



Density
300
250
200
150
100
50
0
120
100
80
60
40
20
0
Time (secs)
1
2
3
4
5
Distance (m)

Part II: Vehicular communication

VANET communications



Communications I

- The main goal is to predict the connectivity of VANETs
- Under common traffic situations.
- Coverage, not range is what is needed (broadcast)
- Some radio details

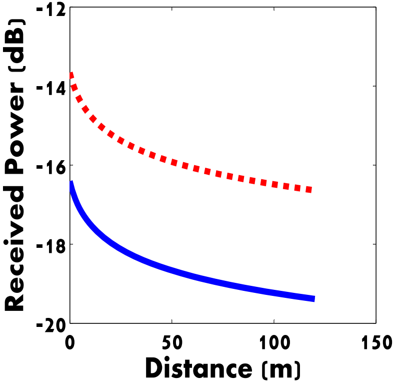
Technology	Subclass	Typical range in meters	Coverage in meters (d)	Transmit power in dBm	Transmit power in mW (P_t)	Transmitter and receiver antenna gains in dBi (G_t, G_r)
Bluetooth	Class 1	10	20	4	2.5	3
802.11a	Outdoor	120	240	27	500	5
802.11p	-	1000	2000	33	2000	5

We want to include the radio connectivity as functions of the density, thus begin with the received power. It is related to the distance between vehicles and is typically given by the equation

$$P_r(dB) = -10\log_{10}\left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 (d+L)^\gamma}\right]$$

P_t is the transmission power G_t is the gain of the transmitter, G_r the gain of the receiver, λ the wavelength of the carrier and importantly d the distance between transmitter and receiver. γ is known as the path loss exponent. A value of two is often chosen for line of sight (LOS) cases between the sender and receiver. In this case of NLOS where vehicles attenuate a broadcast signal, causings a reduction in the coverage. Other works have suggested using values of γ in the range $(2.5, 4.8)$ taken from empirical measurements. L is the length of the vehicle. The antenna is in the middle.

Line of sight/Non line of sight



Information propagation

Divide the application of traffic theory into four distinct phases

1. The level of **connectivity** is defined as the relative time during which adjacent cells are connected.
2. **Reachability** defined as the probability that every two vehicles in the network are connected (clearly coverage dependent)
3. **Broadcast capacity**, defined as the maximum number of successful concurrent transmissions over the road.

Connectivity, reachability and capacity algorithms

- Process a 2-dimensional density matrix in space time.
- Cell "distance" dimension length $\div$ coverage size.
- Cell "time" depends on the road length, the u_{max} and ρ_{max}
- Maximum density is a function of the vehicle length (4m)
- Example for all the vehicles to pass the 2km stretch with our settings of p_{max} and u_{max} we would have approximately 600 time columns.
- A typical run time of 50s will produce 600,000 density values
- In a range a (100, 600) matrix.
- Process the matrix in columns, for each time step $\triangle t_i$.
- $d_{cov}()$ is the cell coverage and $P(r, c)$ as dimensions

Algorithm

1. Normalize P wrt to the maximum density, $\bar{P} = \frac{P}{\rho_{max}}$
2. Obtain the no. of vehicles per cell, $N_i = \bar{P} \cdot \frac{d_{cov}()}{L}$
3. Find nearest whole no. of vehicles, $\tilde{P}$=round(N_i)
4. For each $\triangle t$
   4.1 For active cells (> 1)
   4.2 Do
   4.3 if (all cells are active) then whole network is active at time $\triangle t_i$.
   4.4 else Repeat for all time steps
   4.5 sum to find connected probability.

Notes:

Active cells > 1 network is partially connected

Results

Stop time	Duration (seconds)	Bluetooth coverage	802.11a coverage	Traffic flow	Proportion to to ρ_{max}	Bluetooth coverage	802.11a coverage
Short	30	3%	24%	Light	0.2	4%	26%
Med.	60	6%	32%	Medium	0.5	17%	39%
Long	120	12%	45%	Heavy	0.8	56%	74%

V2V connectivity for traffic behind traffic light and constant initial conditions (right)

Stop time	Duration (seconds)	Bluetooth reach.	802.11a reach.	Traffic flow	Proportion to to ρ_{max}	Bluetooth reach.	802.11a reach.
Short	30	1%	24%	Light	0.2	0%	25%
Medium	60	2%	32%	Medium	0.5	33%	37%
Long	120	10%	45%	Heavy	0.8	70%	73%

V2V reachabilty for step (left) and constant initial conditions (right)

Stop time	Duration (seconds)	Bluetooth broadcast	802.11a broadcast	Traffic flow	Proportion to to ρ_{max}	Bluetooth broadcast	802.11a broadcast
Short	30	0.8%	4%	Light	0.2	1%	15%
Medium	60	1.2%	12%	Medium	0.5	2.5%	27%
Long	120	2%	15%	Heavy	0.8	4%	33%

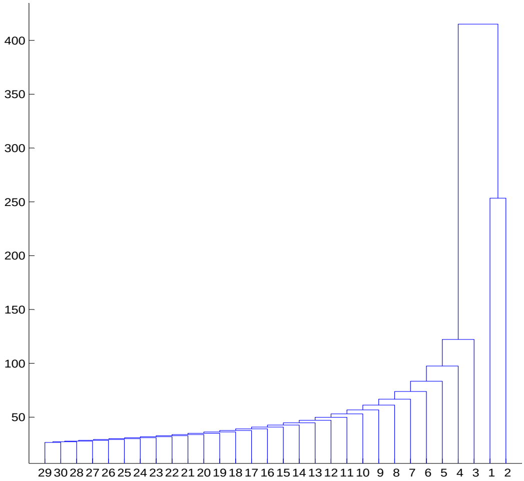
V2V broadcast capacity for step (left) and constant initial conditions (right)

Time to reflect...

- The opposite problem of sparse connectivity from a networking perspective is attenuation from overly dense traffic
- A more involved approach method would be to utilize the local density conditions we derive to estimate the current attenuation.
- *Actually* what we need to do is a cluster analysis on the density wrt to spacetime
- Due to shocks, traffic impediments, human behaviour, etc. the density can change rapidly
- Communications are adaptable, but hints help

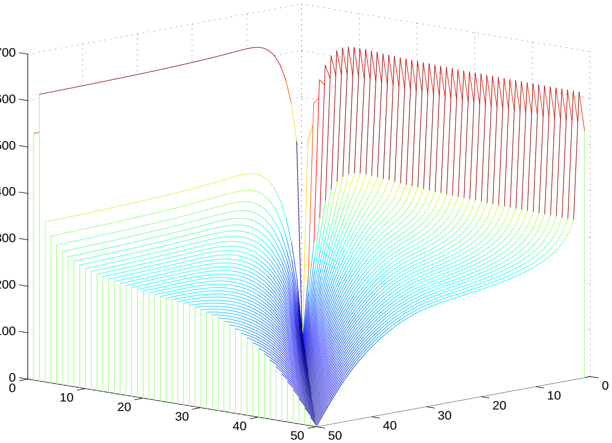
Cluster analysis: dendogram

Is a tree-like representation to show the arrangement of the clusters, hierarchically. Along the x-axis is the leaf nodes and the y-axis is the distance metric between nodes.



Cluster analysis: linkage

- Nearest neighbour (relay)
- Furtheest neighbour (coverage)
- Average neighour



Part III: Extras

Conclusions

- Focus on the application of *traffic flow* theory to VANET
- Different approach to microscopic modeling (plenty)
- Broader goal is to ascertain *coverage* model
- In some way use the "real physical level" to communications applications, i.e. drivers can influence communications
- Need to estimate/measure/model of the initial densities
- Boundary conditions not not covered in this work
- High density due to vehicle shadowing and low density intermittent connectivity
- What is the optimal density (hence spacing) connectivity?
- Derivation of the density can be complex
  - But makes connectivity analysis *much easier*
  - Density $\Leftrightarrow$ Separation $\Leftrightarrow$ Coverage $\Leftrightarrow$ Connectivity

Further work

Maths:

- Glossed over the continuity needs of the traffic variables
- Weak solutions using test functions
- Cluster analysis using distance metrics (VSM?)
- Numerical accuracy?

Vehicular:

- Mixed traffic (bus, trucks)
- Extended to roundabouts, intersections (need to add probablistic aspects)
- Non homogenous cases, traffic leaves and enters a road

$$\frac{\partial \rho}{\partial t} + \frac{dq}{d\rho}\frac{\partial \rho}{\partial x} = \beta \tag{11}$$

- Multilane traffic systems *systems* of non-linear PDEs.

Communications:

- Mixed traffic and radio work (but underway)
- Mate and co. making real measurements
- Optimize power/rate depending on inter-vehicle distances
- Integration with ns-3 (started)

Analysis

- Cluster formation needs more

Related work

- The history of traffic flow lies with fluid dynamicists. Whitam's 1956 publication "Shock waves in the highway" plus his work with Lighthill "On kinematic waves" [Whi56, LW55] showed that shockwaves can be produced in a similar way in traffic flows as in gas and water flows.
- It was later shown that traffic jams displays sharp discontinuities, whereby a correspondence exists with shock waves [Pet72].
- The means were take from road traffic surveys [HBF61].
- In the field of mobile ad-hoc networking Gupta and Kumar present for statically identical randomly located nodes, the throughput obtainable for a source-destination (S-D) pair for a randomly chosen position is $O(\frac{1}{\sqrt{n}})$ [GK00]. Grossglauser and Tse extended this work by including independent node mobility, the average long term throughput per S-D pair can be kept *constant* as the number of nodes per unit area n increases, i.e. $O(n)$ [GT02]. Pishro-Nik et. al provide a general framework to study the fundamental capacity limits of VANETs and show that (indeed) road geometry affects the capacity and connectivity of VANETs and are $\Theta(\frac{1}{n})$ for a single road and $\Theta(\frac{1}{\sqrt{n\ ln(n)}})$ for a grid-like road structure for a S-D pair [Hos07].
- For a detailed analysis on vehicle clustering and connectivity consult [KPD$^+$08].
- Object shadowing exists where vehicles obstruct the path from sender to receiver. interference from overly dense traffic [BVF$^+$10, OBB09]. Experimentally the latter has shown path loss exponents of between 3.3 and 4.8dB at rush hours and exponents of 3.1 and 3.2dB at low traffic periods.

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Guess the location...

huge-highway-full-of-cars.jpg

PDEs

Fundamential theory of calculus

$$\int_a^b q_x(x,t) = q(b,t) - q(a,t)$$

Differentiating an integral

$$\frac{d}{dt}\int_a^b \rho(x,t) = \int_a^b \rho_t(x,t)dx$$

Large box method

Small box method